

#### **Introduction**

Regression can be defined as a functional relationship between two or more correlated variables. It is used to predict one variable given the other. The relationship is usually developed from observed data. The data should be plotted first to see if they appear linear or if at least parts of the data are linear. Linear regression refers to the special class of regression where the relationship between variables forms a straight line.

The linear regression line is of the form y = a + bx, where y is the value of the dependent variable that we are solving for, a is the y intercept, b is the slope, and x is the independent variable.

Linear regression is useful for long-term forecasting of major occurrences and aggregate planning.

The major restriction in using linear regression forecasting is, as the name implies, that past data and future projections are assumed to fall about a straight line.

Linear regression is used both for time series forecasting and for casual relationship forecasting. When the dependent variable (usually the vertical axis on the graph) changes as a result of time (plotted on the horizontal axis), it is time series analysis. When the dependent variable changes because of the change in another variable, this is a casual relationship (such as the demand of cold drinks increasing with the temperature).

## Least Squares Method for Linear Regression

The least squares equation for linear regression is Y = a + bXWhere,

Y = Dependent variable computed by the equation

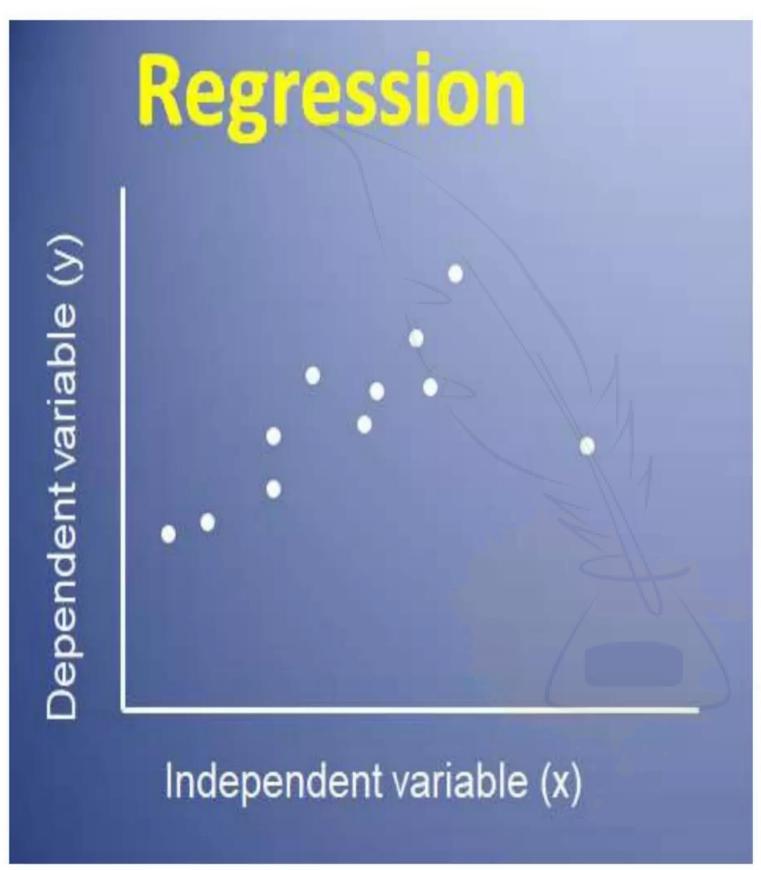
a = y intercept, b = Slope of the line, X = Dependent variable

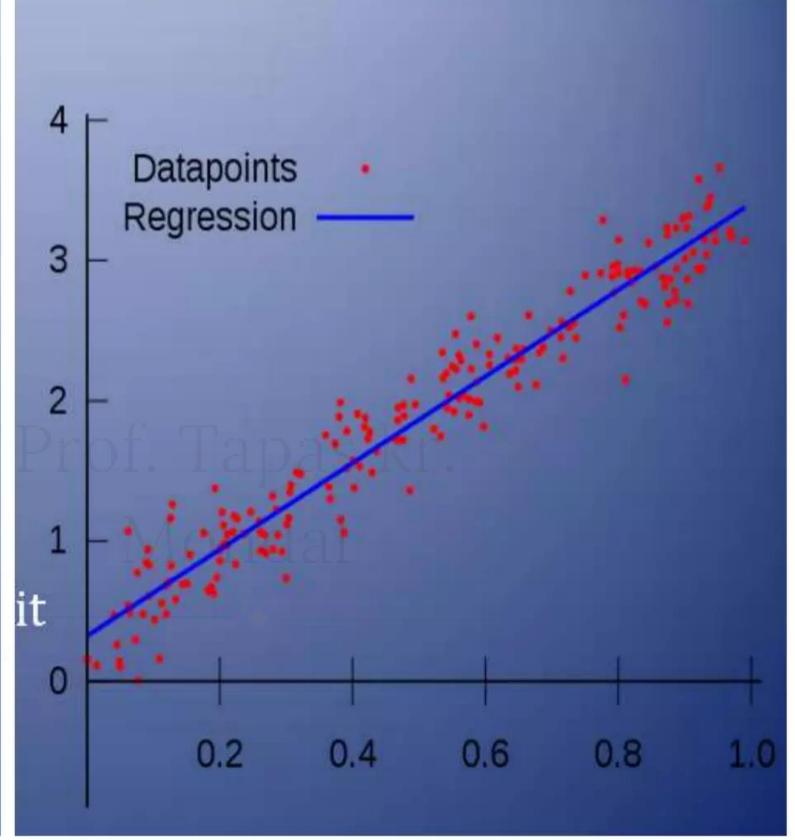
The least squares method tries to fit the line to the data that minimize the sum of the squares of the vertical distance between each data point and its corresponding point on the line.

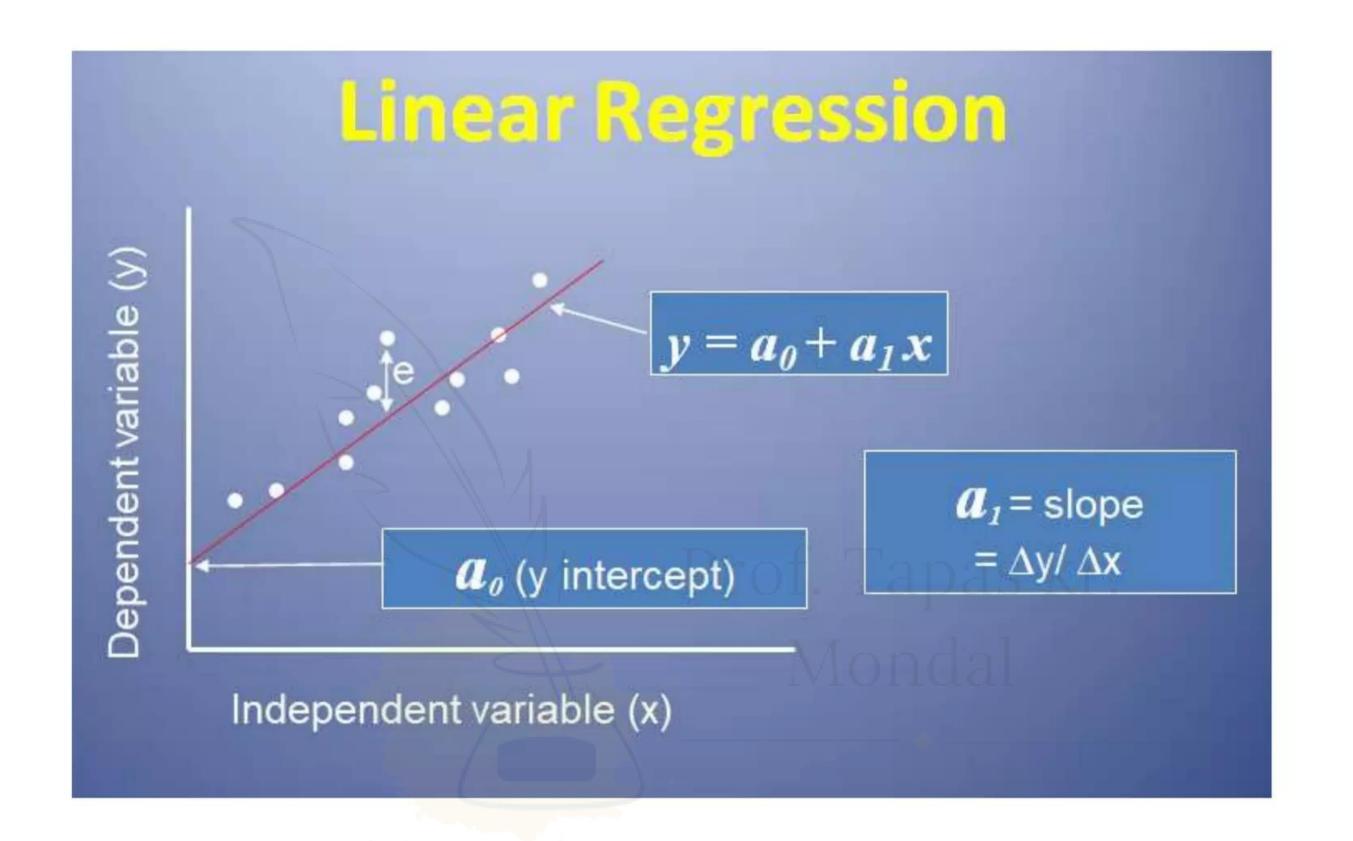
If a straight line is drawn through general area of the points, the difference between the point and the line is y - y'. The sum of the squares of the differences between the plotted data points and the line points is

$$(y_1 - y_1')^2 + (y_2 - y_2')^2 + \dots + (y_{12} - y_{12}')^2$$

The best line to use is the one that minimizes this total.







a<sub>1</sub> is the per unit change in the dependent variable for each unit change in the independent variable.

#### Forecasting

#### Linear Regression Analysis

Maxus Sales Corporation is in the business of selling laptops. They realized the advantages of forecasting very early in their business. They also realized that in order to perform effective forecast, they need to keep track of their current and past sales numbers.

Currently, they are in the process of forecasting their sales numbers for each quarter of the coming year. In order to do this, they have pulled up their sales numbers for the last 12 quarters. These numbers are as follows:

Quarter	Sales	Quarter	Sales
1	600	7	2600
2	1550	V8 O D C S	2900
3	1500	9	3800
4	1500	10	4500
5	2400	11	4000
6	3100	12	4900

Use the Least-squares method to find forecast for the next 4 quarters. Also find out the 'Standard error of the estimate'.

<b>3</b>				
Х	x = (X - m)	У	ху	x <sup>2</sup>
1	-5.5	600	-3300	30.25
~2	-4.5	1550	-6975	20.25
3	-3.5	1500	-5250	12.25
4	-2.5	1500	-3750	6.25
5	-1.5	2400	-3600	2.25
6	-0.5	3100	-1550	0.25
7	0.5	2600	1300	0.25
8	1.5	2900	4350	2.25
9	2.5	3800	9500	6.25
10	3.5	4500	15750	12.25
11	4.5	4000	18000	20.25
12	5.5	4900	26950	30.25
Σ	0	33350	51425	143

$$\sum y = na + b\sum x$$
  
33350 = 12 x a + b x 0  
a = 2779.17

$$\sum xy = a \sum x + b \sum x^2$$

$$y = a + bx$$
  
 $Y = 2779.17 + 359.6 x$   
 $y = 2779.17 + 359.6 (X - m)$   
 $y = 2779.17 + 359.6X - 359.6 (6.5)$ 

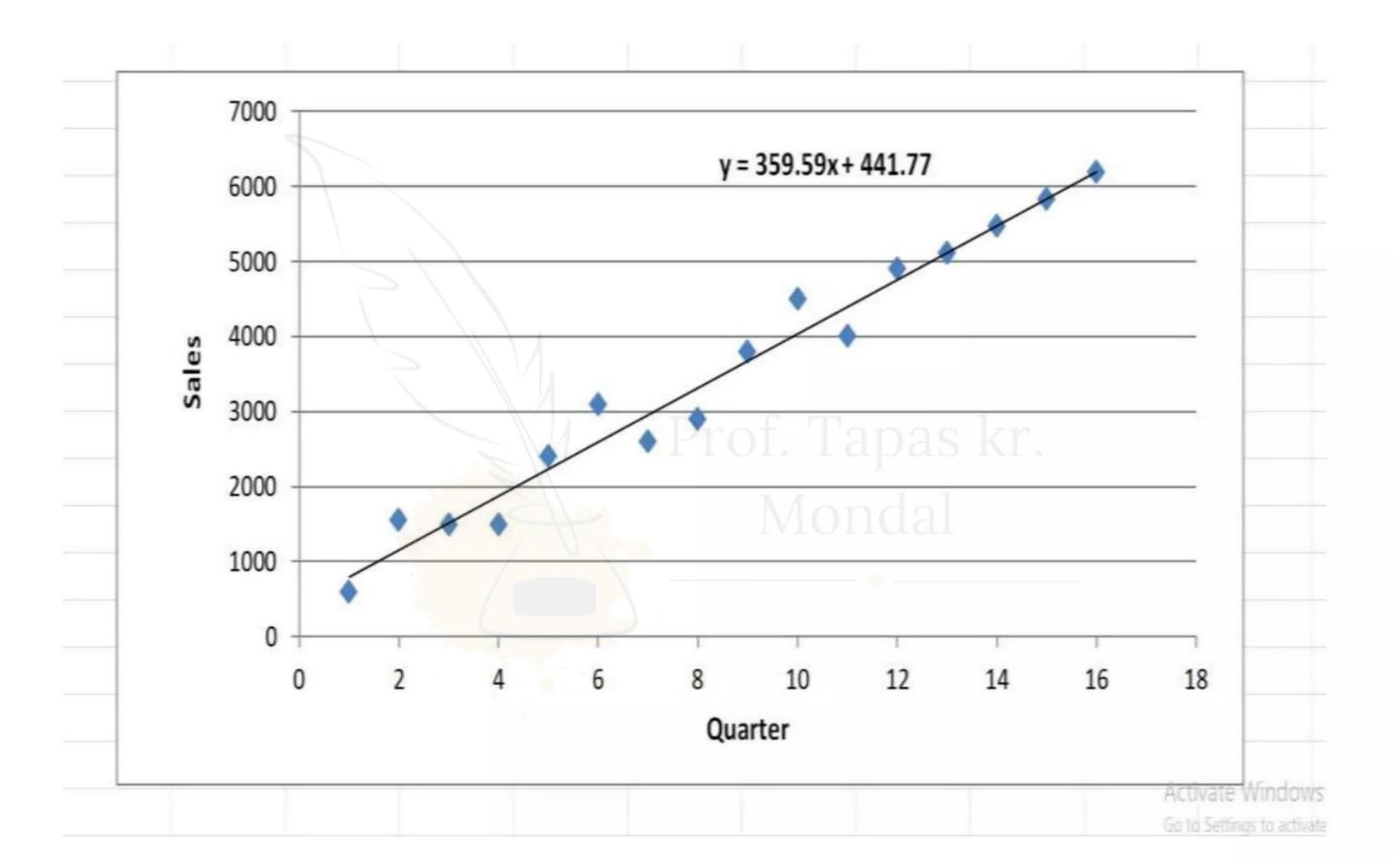
$$y = 441.77 + 359.6 X$$

Where a = 441.77 & b = 359.6, m = mean b = Regression coefficient.

The slope shows that for every unit change in X, Y changes by 359.6

forecasts for periods from 13 to 16 would be

$$y_{13} = 441.77 + 359.6 (13) = 5116.57$$
 $y_{14} = 441.77 + 359.6 (14) = 5476.17$ 
 $y_{15} = 441.77 + 359.6 (15) = 5835.77$ 
 $y_{16} = 441.77 + 359.6 (16) = 6195.37$ 



### **Error Calculation**

$$S_{yx} = \sqrt{\frac{\sum (y_i - y_i')^2}{n - 2}}$$

 $y_i$ = given data value of the dependent variable  $y_i$ '= data value of the dependent variable computed by the equation.

n = number of data points

$$y = 441.77 + 359.6 X$$

$$y_1' = 441.77 + 359.6 (1) = 801.37$$

$$y_2' = 441.77 + 359.6 (2) = 1160.97$$

$$y_3' = 441.77 + 359.6 (3) = 1520.57$$

Similarly calculate  $y_4{}^\prime$  ,  $y_5{}^\prime$  ,  $y_6{}^\prime$  ......  $y_{12}{}^\prime$ 

а	b	x	y'	(y-y') <sup>2</sup>	
441.77	359.6	1	801.37	40549.877	
441.77	359.6	2	1160.97	151344.34	
441.77	359.6	3	1520.57	423.1249	
441.77	359.6	4	1880.17	144529.23	
441.77	359.6	5	2239.77	25673.653	
441.77	359.6	6	2599.37	250630.4	
441.77	359.6	7Pro	2958.97	128859.46	
441.77	359.6	8	3318.57	175200.84	
441.77	359.6	9	3678.17	14842.549	
441.77	359.6	10	4037.77	213656.57	
441.77	359.6	11	4397.37	157902.92	
441.77	359.6	12	4756.97	20457.581	
			Σ	1324070.5	

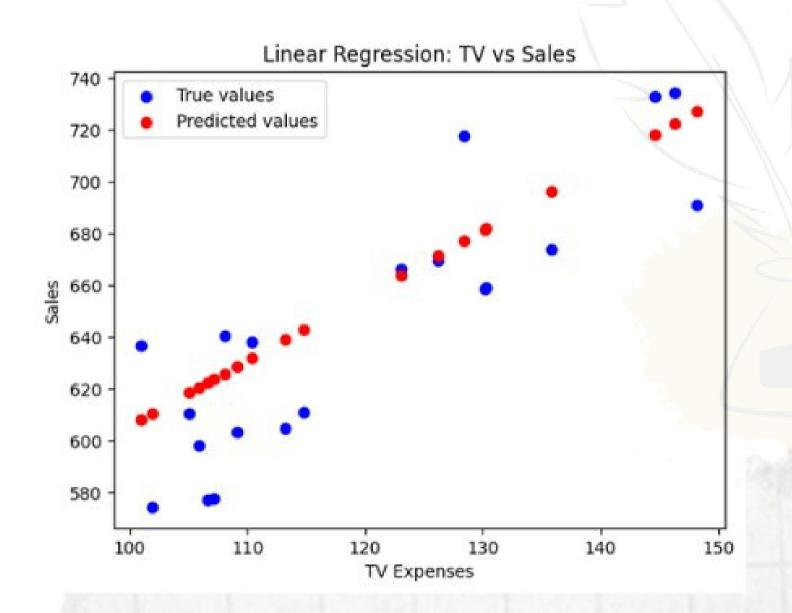
$$S_{yx} = \sqrt{\frac{1324071}{12-2}}$$

$$S_{yx} = \sqrt{132427.921}$$

$$S_{yx} = 363.9$$

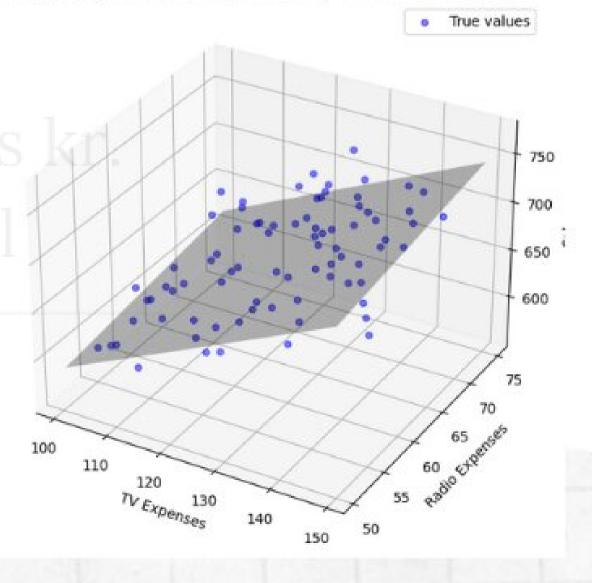
Prof. Tapas kr. Mondal

## LINEAR REGRESSION



# MULTIPLE REGRESION

Multiple Regression: Sales predicted by TV and Radio Expenses





## Multiple Linear Regression (MLR)

['məl-tə-pəl 'li-nē-ər ri-'gre-shən]

A statistical technique that uses several explanatory variables to predict the outcome of a response variable.

Multiple linear regression (MLR), also known simply as multiple regression, is a statistical technique that uses several explanatory variables to predict the outcome of a response variable. The goal of multiple linear regression is to model the linear relationship between the explanatory (independent) variables and response (dependent) variables. In essence, multiple regression is the extension of ordinary least-squares (OLS) regression because it involves more than one explanatory variable.

## Formula and Calculation of Multiple Linear Regression

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \epsilon$$

where, for i = n observations:

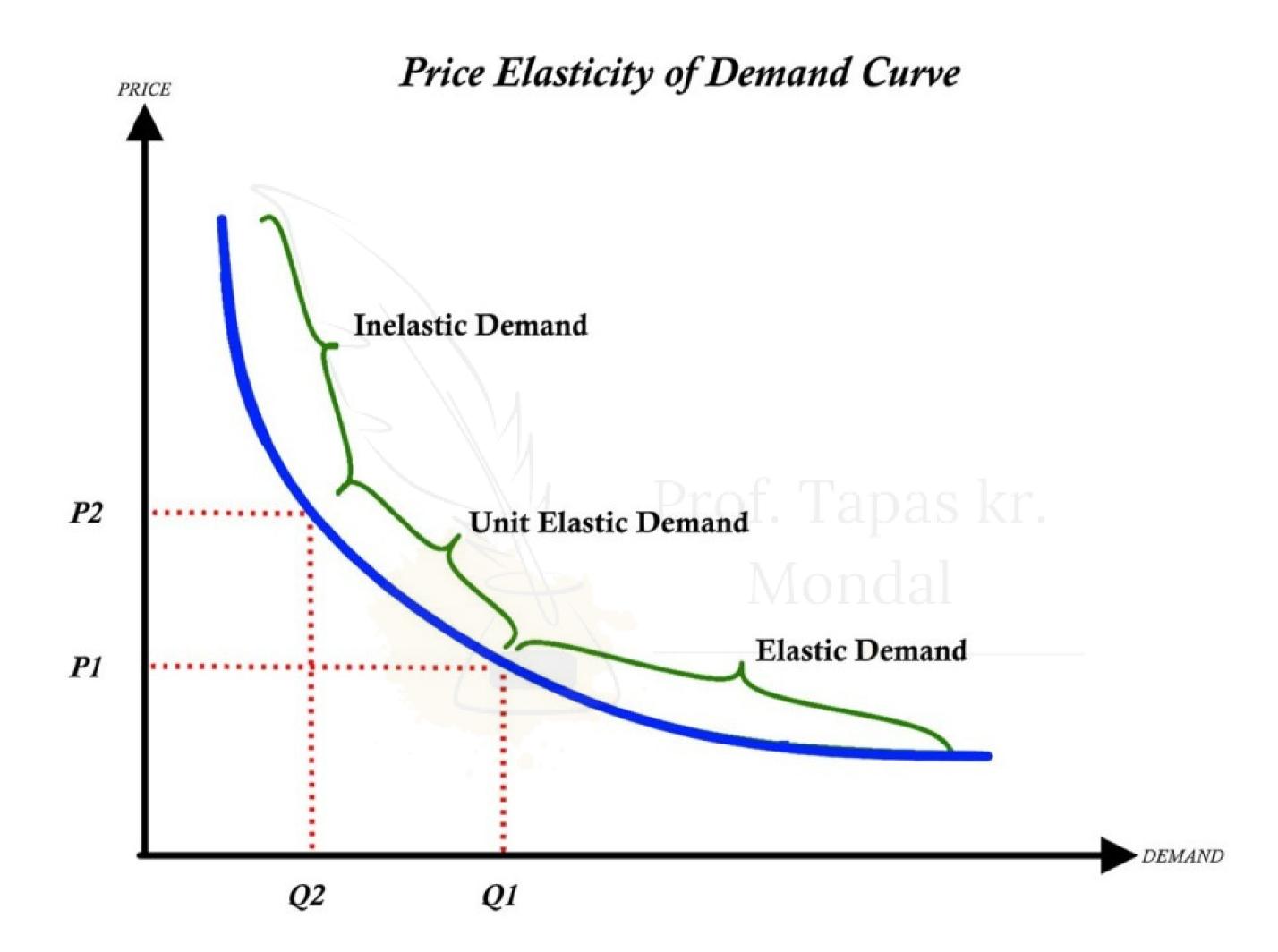
 $y_i = dependent variable$ 

 $x_i = ext{explanatory variables}$  Prof. Tapas kr.

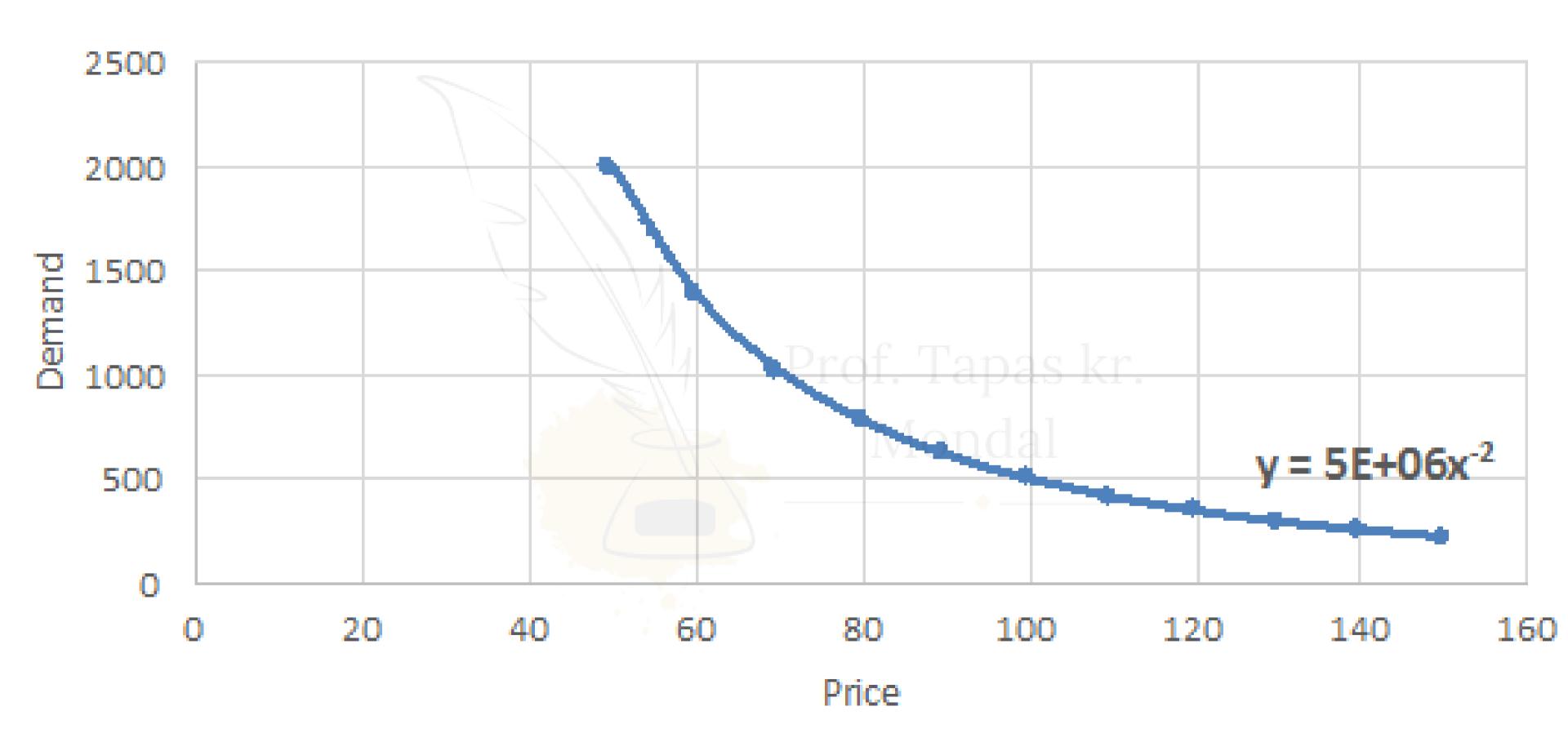
 $\beta_0 = \text{y-intercept (constant term)}$ 

 $\beta_p$  = slope coefficients for each explanatory variable

 $\epsilon$  = the model's error term (also known as the residuals)



#### Power demand curve





Increases average order value



Increases product discovery



Fewer individual orders (saves on shipping, etc.)



Lowers overall margin per product

Greater chance of cost overruns

#### **Cost of Bundle**



#### Cost if Purchased Individually

Product	Price	
McChicken	\$2.99	
Filet O Fish	\$5.69	
Promedium Fries Skr.	\$1.39	
Small Coke	\$1.00	
Total	\$5.38 - \$8.08	

### Mixed Price Bundling – When Does it Work?

... estimates can be derived from surveys, conjoint analysis, etc.

	Customer A Valuation	Customer B Valuation	Customer C Valuation
Beer at ballpark	\$12	\$7	\$11
Hot dog at ball park	\$7	\$13	\$11

#### No Mixed Price Bundle

Consumer surplus = The difference between what someone would have paid vs. what they did pay. As a modeling assumption, We assume consumers will try to maximize their surplus.

PRICE:	Customer A Surplus	Customer B Surplus	Customer C Surplus
Beer Price - \$10	\$2	rof-\$3 apa	as kr\$1
Hot dog Price - \$10	-\$3	\$3	\$1

#### Total revenue

= \$40

Customer A buys Beer @ 10 Customer B buy Hot Dog @ 10 Customer C buys BOTH @ 20

#### With Mixed Price Bundle

	Customer A Surplus	Customer B Surplus	Customer C Surplus
Beer - \$10	\$2	-\$3	\$1
Hot dog - \$10	-\$3	\$3	\$1
Dog N' Brew Special - \$16	\$3	\$4	\$6

Total revenue = \$48

Bundle

### **Pros** of bundle pricing



Long-term revenue growth One-stop shopping

Personalized pricing

Reduced pricing disputes

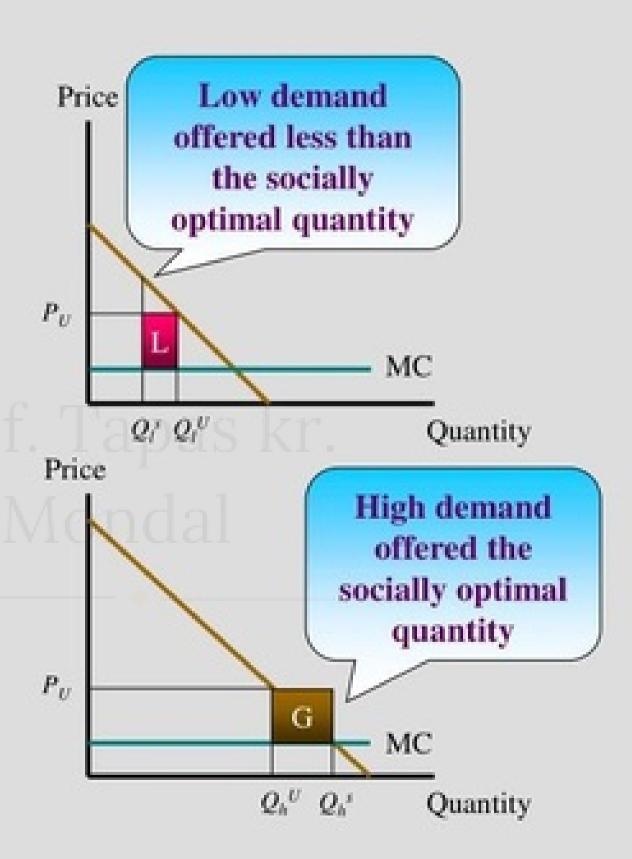
Boost product sales

Partnership opportunities

Cross platform data sharing

### Non-linear pricing and welfare 3

- Menu pricing is less straightforward
  - suppose that there are two markets
    - low demand
    - high demand
- Uniform price is  $P_U$
- Menu pricing gives quantities  $Q_1^s$ ,  $Q_2^s$
- Welfare loss is greater than L
- Welfare gain is less than G





## Price Skimming

['prīs 'ski-min]

When a company charges the highest initial price that customers will pay and then lowers it over time.



### Price skimming strategy



**CONTRIBUTING FACTORS** 

**NEW PRODUCT** 

**BRAND PRESTIGE** 

**FEELING EXCLUSIVE** 

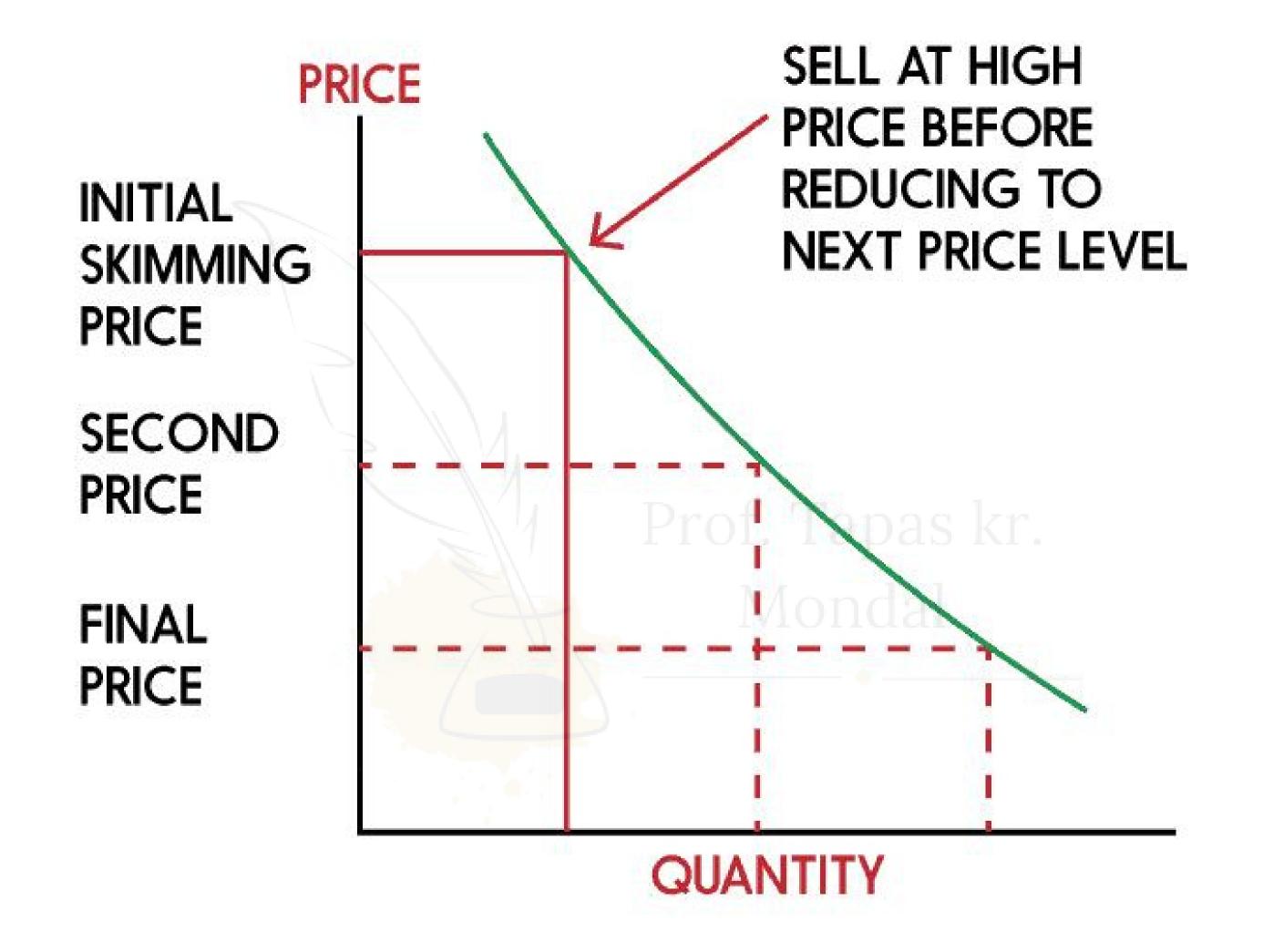
**NEGATIVE MOMENTS** 



**TIME-SENSITIVENESS** 

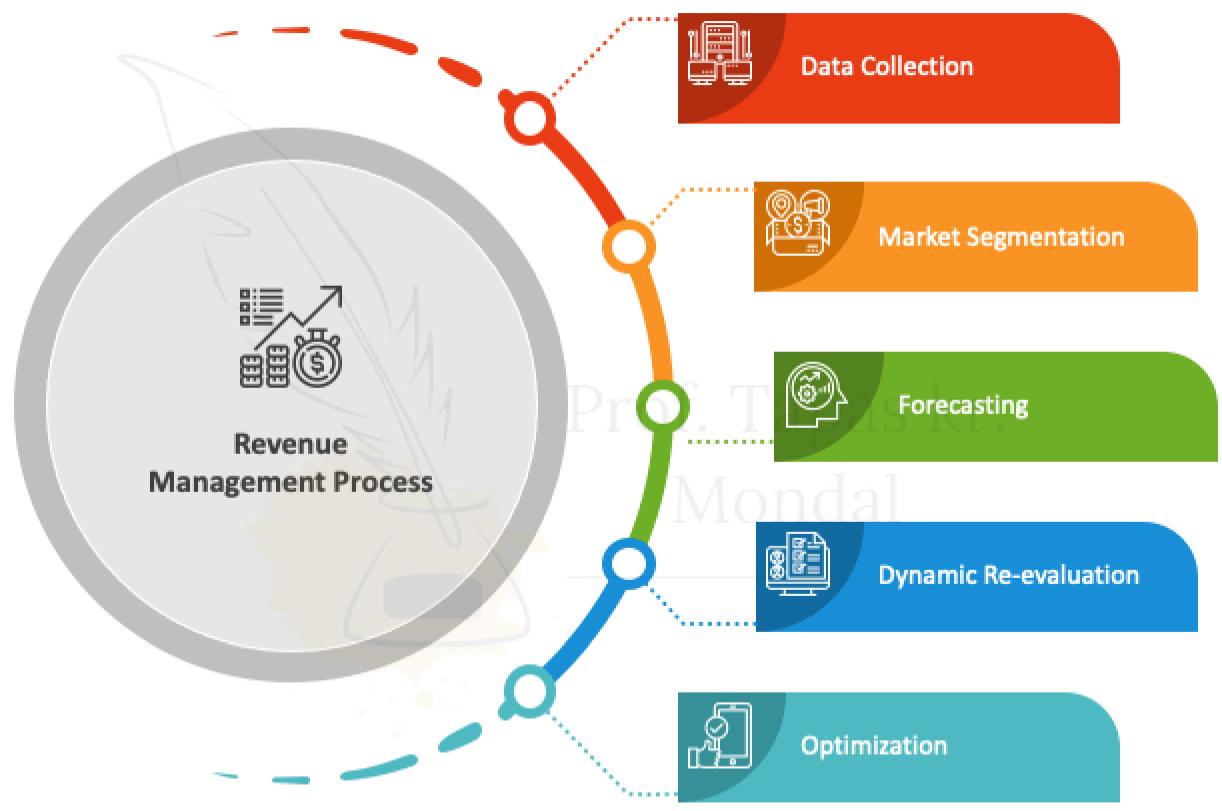
**KEEN COMPETITORS** 

**DISSATISFIED CUSTOMERS** 



#### **REVENUE MANAGEMENT**

Revenue Management Process



#### **4 LEVERS OF REVENUE MANAGEMENT**



#### 1. Pricing Basis

Setting prices by how customer segments perceive specific value to the product or service, rather than based on costs or reacting to competitor prices.



#### 2. Inventory Allocation

Deploying the business's capacity of goods or services depending on demand conditions and prices accepted by customer segments.



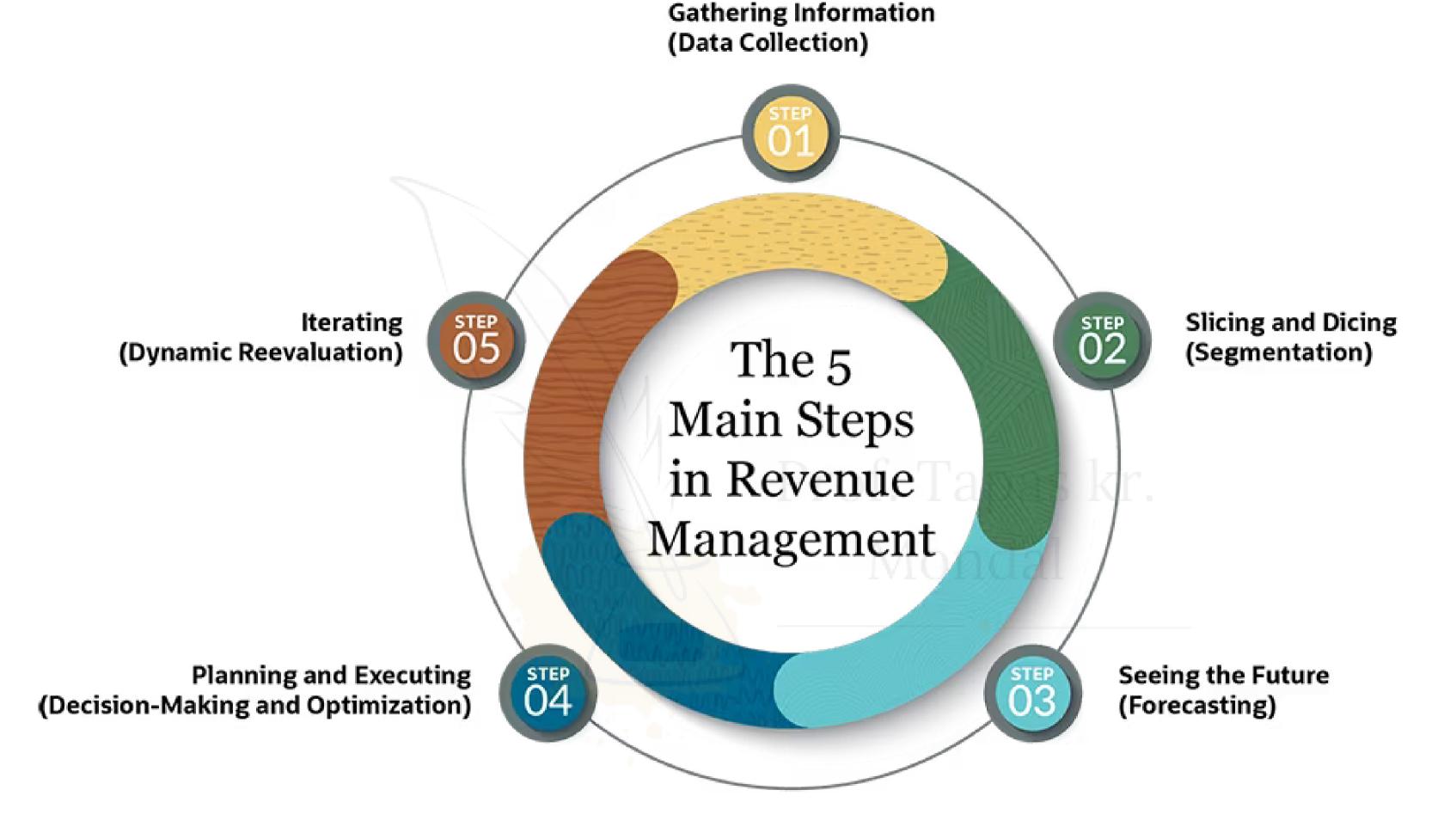
#### 3. Product Configuration

Efficiently customizing or innovating goods and service offerings to meet the expectations of different customer segments while optimizing price points in each segment.



#### 4. Duration Control

Optimizing resource constraints to meet the demand of customer segments by internally managing bottlenecks or externally influencing customer behavior.



Revenue management is a continuously iterative process of gathering information about various customer types, analyzing it to forecast their behavior, offering prices and other terms to maximize revenue by exploiting those behaviors, evaluating the results and making adjustments.

## Thank You

