UNIT 1 INTRODUCTION TO STATISTICAL QUALITY CONTROL

Structure

1.1 Introduction

From the moment we get up until the time we go to bed, we utilize a variety of products and services. For example, we use many things like as toothpaste, soap, detergent, clothing, food, gas stove, automobile, phone, computer, TV, electric bulb, and so on, as well as various services such as water supply, power, transportation, health care, education, and so on.

When we utilize these items and services, we all hope that they meet certain criteria. We say a product or service is of good quality if it meets the requirements required for proper usage. It is deemed to be of low quality if it does not meet the criteria.

In the face of ever-increasing market rivalry, the manufacturer's or producer's primary goal is to accomplish quality assurance in production and service organisations so that his/her product/service can compete in the market. To attain this goal, many statistical techniques for regulating product quality in relation to specified requirements or standards have been created.

Statistical Quality Control (SQC) is a strategy for regulating product quality versus specifications using statistical techniques.

You will learn about the ideas and numerous features of SQC in this unit. In Sec. 1.2, we define the term quality and discuss dimensions of quality, quality control and historical review of quality control. In Sec. 1.3, we describe various aspects of SQC, e.g., the elements and techniques of SQC, statistical process control and product control. We also discuss the causes of variation, which may be due to chance or could be assigned to some factors in the production process in Sec. 1.4. In Sec. 1.5, we introduce the control chart – a tool used in statistical quality control to indicate whether a process is under control or out-of-control. We explain the concept of 3σ limits, different patterns of the control chart and advantages and limitations of SQC in Secs. 1.5 to 1.8. In the next unit, you will study the control charts for variables.

1.1 Objectives

After studying this unit, you should be able to:

- explain the concepts of quality, quality control and statistical quality control (SQC);
- describe the need for statistical quality control;
- distinguish between causes of variation (chance causes and assignable causes) in the production process;
- describe the techniques of SQC;
- define process control and product control;
- explain the concept of control chart, the principle underlying 3σ limits and various control chart patterns; and
- discuss advantages and limitations of SQC.

1.2 Quality

We have just given you a taste of the concept of quality in the unit's introduction. We are now going to define the term "quality." The definition of quality is "degree of excellence." It implies that both items and services must be of high quality. This definition of quality is somewhat subjective because it is based on one's opinion of perfection and changes from scenario to situation or person to person. If you ask 20 different people to define quality, you will most likely get 20 different replies. The best, on the other hand, will be extensively utilized. When asked what quality they prefer, most individuals reply the best. This shows that the general public's perception of the term quality is

favourable. However, observing people's purchasing behaviours, we discover that most individuals choose to buy an item at a discount price during a sale or a mid-priced item rather than the costliest one. This suggests that when purchasing something, individuals want the finest that money can buy. Aside from pricing, there are many other factors to consider for quality, such as cost, size, performance, warranty, look, and so on.

The idea of quality as goodness, the best, or luxury is insufficient for quality control specialists since the best is determined by an individual's viewpoint. For example, a person may believe that a car is a fine vehicle for Delhi roads but a terrible vehicle for the woods of Himachal Pradesh, Madhya Pradesh, and so on, because the ideal vehicle for these locations is one with four-wheel drive. Similarly, a decent walking shoe is ideal for strolling but not for running. As a result, the intended use of the product or service must be incorporated into the quality concept. As a result, we must define the term "quality."

1.2.1 Defining Quality

There are several ways of defining quality:

1. Initially, quality was defined as conforming to criteria. This means that any product should be manufactured in compliance with the specifications supplied. If a product achieves its criteria, it is said to be of high quality. There are several problems with this definition. In certain cases, even if a product fits all of the criteria, its utility is insufficient. Assume a buyer wants to buy a touch-screen phone with amazing sound. The seller shows him/her such a mobile set. The user, on the other hand, may consider that the mobile phone is excessively huge. As a result, he or she may be unable to obtain it since it is unsuitable for his or her requirements.

   Therefore, from the point of view of customers, such products are not useful. Thus, there is a need to redefine the term quality.

2. The concept of quality has been broadened to encompass suitability for purpose. This means that the generated product must be fit for usage and meet its specifications. However, it was observed that in certain cases, even though a product met all criteria and was fit for use, it could not be sold because it did not appeal to the consumer. As a result, in order to sell any product, the customer's perspective must be taken into account.

3. The third definition of quality was customer pleasure. This signifies that a product that satisfies the consumer qualifies as a quality product. Then it was discovered that each buyer would have several requests in relation to each thing that he or she desired to purchase. Customers, on the other hand, seldom convey all of their expectations. Let's say a person goes to a restaurant and orders a pizza. That individual would be quite upset if the pizza he was given was not heated. However, it is also true that the consumer would never express a desire for a heated pizza! Many times, clients are dissatisfied even though all of their acknowledged or stated requirements are met. Hence, while defining quality, there is a need for considering unexpressed or unstated needs of customers.

4. The fourth criteria of quality provided was satisfying the consumer. Delight comes before fulfilment. When a product meets the customer's declared and unexpressed demands, he or she is overjoyed. This definition, however, has been improved.

5. Enchanting consumers was the fifth term of excellence presented. According to this concept, the maker has two roles: first, he or she must understand the needs of the clients. Second, he or she should make clients aware of this truth while also convincing them that these are the things they desire. For example, an electric bulb producer must educate clients about LEDs and persuade them to choose LEDs since they consume less power, even though they are somewhat more expensive. Every manufacturer is now expected to adhere to these criteria.

From the above discussion, we may conclude that in the manufacturing and services sector, the following aspects have to be incorporated in the definition of quality:

a. conforming to specifications,

b. fitness for use,

c. customer satisfaction,

d. delighting the customer, and

e. enchanting the customers.

Having explained the concept of quality in industry, and defined it we now describe various dimensions of quality.

1.2.2 Dimensions of Quality

In the previous section, we have explained the concept of quality and defined it. In 1988, David A. Garvin summarized eight basic elements of quality, which are known as the dimensions of quality. We describe them briefly.

i. **Performance:** The first quality dimension is performance. It refers to a product's basic operational features. Consumers evaluate the quality of a product based on its performance after comparing it to the products of competitors or the market standard. A cell phone, for example, may be rated based on its sound quality, weight, size, functionalities, and so on. Similarly, the pick-up, fuel efficiency, and other characteristics of a motorbike may be used to evaluate it.

ii. **Features:** The second dimension of quality is features. These are supplementary features that are accessible in products in addition to the core functioning qualities. For example, complimentary beverages and snacks on a flight or at a hotel, Bluetooth and FM radio on a mobile device, and so on. This quality component also influences a customer's decision.

iii. **Reliability:** The chance of a product failing within a particular time period is referred to as reliability. When a product fails regularly, we refer to it as unreliable. For example, if a specific company's television requires regular maintenance, we call it unreliable. Customers' perceptions of quality are impacted by the reliability of various items, including computers, televisions, and vehicles.

iv. **Conformance (Agreement):** Conformance is defined as fulfilling requirements. Customers understandably want the product to satisfy its specs. When we buy a motorbike, for example, we look to see if the seating space, weight, size, pick-up, fuel efficiency, and so on meet the company's criteria. This factor also impacts consumer selection.

v. **Durability:** Durability is a measure of product longevity. It may also be thought of as the product's operational life, or how long it can be utilised before needing to be replaced. The life of an electric bulb is an example of durability: when the filament burns out, the bulb must be replaced because repair is not feasible at this point. Customers' perceptions of durability are also altered.

vi. **Serviceability:** The sixth component of quality is serviceability. Consumers are concerned not just about product failure, but also about the time it takes for the product to be repaired and restored. The ease with which a product may be maintained and returned to functioning mode is referred to as serviceability. Assume a buyer wants to purchase a washing machine. When settling on a firm, he or she should evaluate how long it takes the company to service or repair it if it breaks down.

vii. **Aesthetics:** Aesthetics, or how a thing appears, sounds, feels, and so on, is the seventh dimension of quality. A product's aesthetic value is entirely subjective. Some people may find a certain automobile visually pleasing, while others may not.

viii. **Reputation:** The company's reputation is linked to its historical performance. Customers frequently inquire about the quality of previous items manufactured by the organisation. For example, when a firm debuts a new automobile, customers frequently expect that the new model will be successful because the company's previous cars performed well.

In order to design and manufacture products of high quality, it is necessary to incorporate all eight dimensions of quality.

You may like to pause here and check your understanding about the definition of quality and its dimensions by answering the following exercise.

E1) Choose the correct option from the following:

i) Quality means

a) fitness for use

b) degree of excellence

c) conformance to requirement or specifications

d) all of the above.

ii) Primary operating characteristic of a product is known as

a) aesthetics

b) reliability

c) performance

d) features

iii) Durability refers to a measure of

a) product life

b) specifications

c) past performance of the product

d) the probability of a product's failure within a specified time period.

So far you have studied the definition of quality and its dimensions. We now introduce the concept of quality control.

1.2.3 Quality Control

We first understand what we mean by control.

The process/procedure/method that is applied to meet the specifications or standard is known as control. Process control works on feedback and comprises the following steps:

- Choose the parameter to control, i.e., we first choose the characteristic that we intend to control such as length, height, weight, defects, etc.
- Choose the unit of measurement, e.g., centimetre (cm), millimetre (mm), gram (g), etc.
- Set the standard or goal for the parameter to control, e.g., 5 cm, 10 g, etc.
- Select a sensing device, which can measure the parameter to control in terms of the unit of measure, e.g., scale, a weighing balance, etc.
- Measure the actual performance.
- Compare the actual performance with the standard.

- Take necessary action when there is a difference between actual performance and the standard.

Thus, quality control can be defined as:

"The process of measuring the quality characteristics of a product, comparing them to specifications or standards, and taking appropriate action whenever there is a difference between actual quality and specifications or standards."

Quality is now controlled using statistical methods. This is known as Statistical Quality Control, which we will discuss in the next section. However, before you go any further, you might be interested in learning how the notion of quality control evolved historically.

1.2.4 Historical Review of Quality Control

Quality control is not a new concept. We may claim it started when humans lived in forests and ate raw vegetation and animals. They employed natural resources at the time and may have encountered issues such as determining which plants were edible and which were toxic to health. Humans began processing natural materials with the emergence of early technologies. They created things and tested them to see if they performed effectively for their intended purpose. They made rough-hewn stone tools, for example. They could have examined the tool's tip to determine if it was sharp enough for their needs. Humans were both makers and consumers during the time. This scenario evolved over time.

As civilisations evolved, jobs were split and people specialised in the production of various commodities and services. Farmers, for example, grew food, potters manufactured pots, and weavers woven the fabric that everyone needed. This specialisation established a barrier between the manufacturer and the user. As a result, the producer/manufacturer and the user were no longer the same person.

However, the goods were not overly sophisticated, and the consumer had extensive prior experience with the merchandise. As a result, the user could be confident that the items were safe to use. With the onset of industrialization, the situation altered dramatically, and the concept of quality control experienced a sea shift.

For the sake of interest, we describe below the important stages in the historical evolution of quality control:

a. **Operator or craftsman quality control:** Quality control was ensured/observed by a single worker (operator or artisan) or a very limited group of workers who completely controlled the quality of their work from early civilisation until the industrial revolution. This was the case until the eighteenth century.
b. **Foreman or supervisor control**: large factories were built following the industrial revolution to accommodate rising consumer demand. As a result, the number of workers in the industries rose. As a result, supervisors were required to lead, control, and administrate other personnel. Generally, the supervisor was chosen from among the industrial workers who had a solid understanding of the task. The supervisor tasks also included checking the quality of work done by personnel. This scenario persisted until the outbreak of World War I.
c. **Inspector of quality control**: Because of replaceable components on a mass scale, the production system got more complicated, and it became necessary to closely analyse each component using measurement tools. Obviously, the supervisor could not do this since he or she was only knowledgeable about one aspect of the product. As a result, a full-time inspector was dispatched to check the final items, and quality control was separated from manufacturing.
d. **Statistical quality control:** Initially, all items were examined by quality control inspectors. However, this was both expensive and time intensive. Walter A. Shewhart, a researcher at Bell Telephone Laboratories (a

Research and Development Unit of the American Telephone and Telegraph Company), created the statistical control chart approach for controlling product variability in 1924. This is commonly seen as the start of statistical quality control (SQC).

In the same decade, Bell Telephone Laboratories researchers Harold F. Dodge and Harry G. Roming developed statistically based acceptance sampling as an alternative to 100% inspection.

The control chart and acceptability sample approaches were utilised in the Bell system, although neither was extensively used outside of it. This, however, altered during World War II. The US Army's ordinance department was faced with the difficulty of obtaining enormous amounts of weaponry and ammunition from different vendors at acceptable levels of quality. In 1942, the War Department formed a quality control unit, and acceptance sampling became widely employed.

Japan was attempting to recover from the damage of World conflict II after the conflict. Japanese industries were nearly decimated, and its leaders understood that restoring the industry was critical to the nation's existence. Japanese businessmen agreed to hire an American statistician as a consultant. W. Edwards Deming, who had studied under Walter Shewhart, was the guy they picked. Deming created theories on how to utilise statistics to enhance industrial quality. In Japan, he conducted a series of seminars on statistical quality improvement approaches. Japan embraced these ideas and became one of the world's most successful industrial countries within a few decades.

So far, we have given you an overview of the concepts of quality and quality control. We now discuss statistical quality control.

1.3 Statistical Quality Control (SQC)

The primary goal of manufacturers or producers in today's highly competitive market is to attain quality assurance in production and service companies. In order to attain this goal, many statistical techniques for controlling product quality in relation to requirements or standards have been created. Statistical Quality Control (SQC) refers to the process of applying statistical methods to control product quality in relation to requirements.

Statistical quality control is defined as the technique of applying statistical methods based on the theory of probability and sampling to establish quality standard and to maintain it in the most economical manner.

Let us now outline the elements that constitute SQC.

1.3.1 Elements of SQC

The following are the main elements of SQC:

a. **Sample Inspection:** We understand that 100% inspection necessitates a significant investment of time, money, manpower, and resources. Furthermore, if the nature of the product is such that it is fully destroyed during the inspection process, such as a bulb, candle, ammunition, food, and so on, 100% inspection is not feasible. SQC is thus based on sample inspection. The sampling inspection approach involves randomly selecting some things or units (called samples) from the process and inspecting each unit of the sample.
b. **Use of Statistical Methods:** Random sampling, mean, range, standard deviation, mean deviation, standard error, and concepts such as probability, binomial distribution, Poisson distribution, normal distribution, and so on are employed in SQC. Because this form of quality control makes considerable use of statistics, it is known as Statistical Quality Control.

c. **Fundamental Objectives:** The primary goal of SQC is to determine whether or not the unit produced meets its standards. If the unit produced does not meet its standards and there is a variance in quality, it is vital to discover the sources of variation and, if feasible, eradicate them.
d. **Decision Making:** We use SQC to determine if the product's quality or the process of manufacturing/producing things is under control.
e. **Specifications, Production and Inspection:** The SQC approach aids in the specification, manufacture, and inspection of a product.

We now describe the techniques of statistical quality control.

1.3.2 Techniques of SQC

The important techniques used for statistical quality control can be broadly classified into two categories:

- Statistical Process Control (SPC) or simply Process Control, and
- Product Control.

These techniques are further classified into different categories as shown in Fig.1.1.

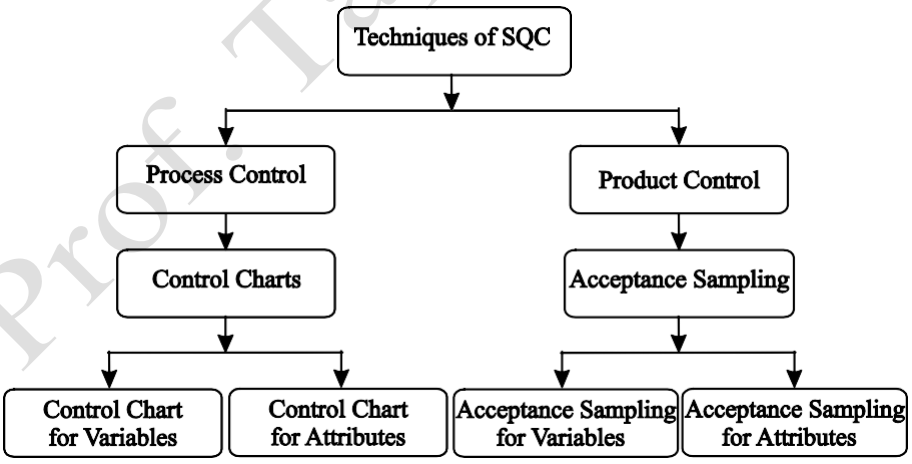

Fig. 1.1: Classification of SQC techniques.

Let us discuss both categories of SQC techniques in some detail.

1.3.3 Statistical Process Control (SPC)

The first component of SQC is statistical process control (SPC), often known as process control (PC). To comprehend SPC, we must first grasp the notion of process in quality control.

A process is a sequence of processes or events that converts input to output. If the final output product meets the stated requirements or standard quality, it is considered to be stable or repeatable. However, the steady process can be disrupted by a variety of factors such as low raw material quality, changes in machine settings, the employment of untrained labour, an inefficient machine, and so on. In such cases, we need a tool or approach to help us regulate the process. This is referred to as statistical process control (SPC).

Statistical process control is a technique for understanding and monitoring a process by periodically collecting data on quality characteristics from the process, analysing them, and taking appropriate actions whenever there is a difference between actual quality and the specifications or standard.

The statistical process control approach is widely utilised in practically all industrial processes to achieve process stability and continual product quality improvement. Its primary tools are:

- Histogram
- Check sheet
- Pareto chart
- Cause and effect diagram
- Process flow diagram
- Scatter diagram
- Control chart

We will only present the "control chart" approach described by W.A. Shewhart in 1924 because it is the most often used technique today. In reality, it is most likely an excellent approach for quality control and improvement. In Section 1.5, we cover the fundamentals of the control chart. We explore many forms of control charts in Units 2, 3, and 4 of this chapter.

1.3.4 Product Control

In many cases, the product is so complicated that a manufacturer cannot produce all of its components or elements. As a result, one or more components of a product are obtained from third-party agents or suppliers, and the manufacturer has no direct control over the quality of such component (s). In such cases, the producer is faced with the challenge of regulating the quality of components acquired from abroad. Furthermore, the maker must maintain quality control over the finished product. As a result, he or she must also ensure that the finished product fulfils standards and that different batches of the product do not contain an abnormally large number of faulty pieces. Such issues are under the purview of product control.

Product control means to control the products in such a way that these are free from defects and conform to their specifications.

Initially, 100% inspection was used for product control, which meant that each and every item manufactured or received from outside sources was inspected. This method of inspection has the advantage of providing total confidence that all faulty units in the tested batch have been removed. It is, however, time consuming and pricey. Furthermore, if a product is destroyed during inspection, such as a light bulb, crackers, ammo, or the image tube of a television, 100% inspection is not feasible.

As an alternative to 100% inspection, sample inspection or acceptance sampling was created.

Acceptance sampling is a process in which a small portion or fraction of items/units are chosen at random from a lot and evaluated to determine whether the lot should be accepted or rejected based on the information acquired from sample inspection. Acceptance sampling is used to accomplish product control in this manner.

There are several methods of acceptance sampling. We'll go through it in depth in Unit 5.

You might want to take a break now to double-check your comprehension of the many features of SQC covered in this part. Complete the following exercises.

E2) Choose the correct option from the following:

i) Statistical quality control (SQC) is a technique of
   a) process control
   b) product control
   c) both (a) and (b)

ii) The statistical techniques used in statistical quality control are
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

iii) Process control is achieved through
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

iv) Product control is achieved through
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

You have learnt that SQC techniques are used for process control and product control so that the products of desired quality may be manufactured. We now explain the causes of variation in the process.

1.4 Causes of Variation

Variation in produced items is unavoidable; it is a feature of life in both nature and industry. Even if a manufacturing process is perfectly conceived and meticulously managed, no two items are similar. Depending on the sources of variation, the difference between any two items might be extremely great, moderate, very little, or even undetectable. For example, the weight of a specific type of vehicle varies from unit to unit, the weight of milk packets may fluctuate very slightly from one another, the length of ball pen refills, the diameter of cricket balls may also differ, and so on. The presence of diversity in items has an impact on quality. So, the aim of SQC is to trace the sources of such variation and try to eliminate them as far as possible. The causes of variation are broadly classified into two categories:

- Chance causes, and
- Assignable causes.

1.4.1 Chance Causes

Chance causes are sometimes referred to as random, natural, or common causes. Even in a well-designed or meticulously managed manufacturing process, product variability arises owing to natural/random factors. Even if the process is run under the identical conditions, that is, with the same raw material quality and no changes in machine settings, operators, or the environment, there is a distinct pattern of variability in the result.

For example, the diameter of ball bearings varies slightly, the weight of cricket balls changes slightly, the fuel economy of a certain type of vehicle varies slightly, and so on. Such variability is caused by several common or accidental reasons, which may have a minimal impact on the process output. These are referred to as random causes of variance. These may occur as a result of the inflexibility of older machinery, variations in bought material, bad illumination, the level of worker training, or other less evident causes. These may or may not be present at the same moment, but when they are, the consequences are random.

If the output quality fluctuates too much owing to random factors, the process must be altered or adjusted to eliminate one or more of these factors. Because process reform or modification is the job of management, eliminating common/chance sources of variation is generally the task of management rather than employees. It may be impossible to exclude all potential factors in a process. Even though variance due to random reasons exists in the manufacturing process, it is still said to be under statistical control.

1.4.2 Assignable Causes

Another type of variability may be found in the output of a process on occasion. Variability of this type is caused by changes in raw materials, equipment, operator, environment, or any other component of the process, rather than by changes in the process design. These are known as assignable causes, as well as unusual, non-random, or unnatural causes. Assignable reasons include an accidently incorrect machine setup, a worker becoming unwell and continuing to work, a change of operators or shift, breakages, misreading of scales, a batch of bad raw material, and so on.

Because the influence of assignable causes is localised inside a process, employees or their immediate supervisor can eradicate them. Variability due to assignable reasons is typically more than variability due to chance factors, and it typically reflects an undesirable level of process performance. An out-of-control process is one that is running in the presence of assignable causes.

You can now check your understanding of the causes of variation by answering the following exercise.

E3) A company manufactures cricket balls. The statistical quality controllerof the company finds that there is a variation in the weight of cricket balls. Answers the following:

a) The variation in the weight of cricket balls may be due to
   a. assignable causes
   b. chance causes
   c. both (a) and (b)

b) If the variation in the weight is due to chance causes, it is
   a. controllable
   b. not controllable
   c. both (a) and (b)

c) If the variation in the weight is due to assignable causes, it is
   a. controllable
   b. not controllable
   c. both (a) and (b)

d) The variation due to chance causes

a. is tolerable
b. does not affecting the quality of a product
c. is uncontrollable
d. all of the above

e) The variation due to assignable causes
a. can be removed
b. cannot be removed
c. can be removed sometimes
d. can be removed most of the times.

Having explained the causes of variations in the production process, we now introduce the technique of control chart. It is the main technique used for process control.

1.5 Control Charts

You would agree that one of the most sensitive statistical instruments is graphical representation. As a result, we may visually display the quality characteristics of the final product, such as weight, length, diameter, flaws, and so on, to comprehend, describe, or monitor process variation. Walter A. Shewhart pioneered the concept of visually portraying quality attributes. He devised control charts for industrial processes to differentiate between acceptable (chance) variance and assignable variation. He discovered that using control charts allowed him to find assignable causes of variance rapidly and take remedial action to eradicate them.

A control chart is a two-dimensional graphical representation of a quality feature measured or computed from samples in terms of mean or other statistic and plotted against the sample number or time at which the sample is obtained from the process.

The control chart idea is based on sampling and probability theory. A sample statistic of a quality feature, such as mean, range, proportion of faulty units, and so on, is taken along the Y-axis of a control chart, while the sample number or time is taken along the X-axis. A control chart is made up of three horizontal lines, as illustrated below:

a. **Central Line (CL):** The value represented by the control chart's centre line might have three distinct meanings based on the available data. First, it might be the quality characteristic's average value or the plotted points' average. Second, it might be a reference or standard value based on representative historical data, or it can be an intended (targeted) value based on specifications. Third, if the population parameter's value is known, it can be used. A solid line is commonly used to indicate the central line.
b. **Upper Control Line:** The top control line denotes the maximum variance in the quality characteristic. As a result, this line is known as the upper control limit (UCL). The UCL is typically represented with a dotted line.
c. **Lower Control Line:** The lower control line reflects the variance in the quality feature with the lowest value. As a result, this line is known as the lower control limit (LCL). The LCL is typically represented by a dotted line.

The UCL and LCL, like the centre line, have three interpretations based on the given data. These limits are derived by the use of the idea of 3 (three sigma) limits, which we will discuss in Sec. 1.6.

We take samples of a few units/items from the process at regular intervals to create control charts. Then, for each unit of the selected samples, we measure the quality characteristic, such as length of foil, diameter of ball bearing, weight of cricket ball, and so on, or count the number of flaws, number of faulty units, and so on. Following that, we compute

statistics for each sample, such as mean, standard deviation, range, fraction of faulty units, and so on. The derived statistic's values are then plotted against the sample number or time. A line can connect the sample points on the graph. It is not necessary to connect the sample points with line segments, but doing so allows us to clearly grasp their sequence (pattern) across time.

The control chart demonstrates that the process is statistically controlled if all sample points are on or between the top and lower control boundaries. That is, the process has purely random causes. There is no assignable cause in the procedure. If one or more sample points are beyond the control boundaries, the control chart warns (indicates) that the process is not statistically controlled. The procedure has some assignable causes. To put the process under statistical control, the assignable causes must be investigated and remedial action taken to eradicate them.

However, a control chart cannot tell us what is wrong it. In fact, it is the responsibility of the supervisor or quality control manager to find out what has gone wrong.

Note that the sample points may be inside the control boundaries but act in a systematic or non-random manner at times. This is also an indicator that the process has gotten out of hand. Assume that 22 of the previous 25 sample points are below the centre line but above the lower control line, and just three are above the centre line but below the upper control line. Because it is not random in appearance, this pattern suggests that the process is not statistically controlled. This topic will be covered in further depth in Section 1.7.

You may like try the following exercise before studying further.

E4) Choose the correct option from the following:

i) Control chart is a
   a) one-dimensional chart.
   b) two-dimensional chart.
   c) three-dimensional chart.
   d) none of the above.

ii) Control chart consists of
   a) one control line.
   b) two control lines.
   c) three control lines.
   d) four control lines.

iii) If one or more sample points lie outside the control limits, the control chart indicates that
   a) there is no assignable cause in the process and the process is under statistical control.
   b) there is at least one assignable cause in the process and the process is under statistical control.
   c) there is at least one assignable cause in the process and the process is out of statistical control.
   d) there is no assignable cause in the process and the process is out of statistical control.

iv) Control charts in statistical quality control are used for.
   a) describing the pattern of variation.
   b) checking whether the variability in the product is within the tolerance limits or not.
   c) Both (a) & (b)

In Sec. 1.5, you have learnt about the control chart, which contains a centre line (CL), lower control limit (LCL) and upper control limit (UCL). We now discuss how to obtain the centre line and control limits for a control chart. The UCL and LCL are also called 3σ limits.

1.6 3σ LIMITS

A probability distribution or a frequency distribution can be used to characterise the quality feature. In the majority of cases, a quality feature follows or may be approximated by a normal distribution. You studied the normal distribution in chance Theory Unit 14 of MST-003, and you know that the chance that a normally distributed random variable (X) lies between 3 and 3 is 0.9973, where and are the mean and standard deviation of the random variable (X).

Thus,

$$P[\mu - 3\sigma \le X \le \mu + 3\sigma] = 0.9973$$

So the probability that the random variable X lies outside the limits $\mu \pm 3\sigma$ is 1– 0.9973 = 0.0027, which is very small. It means that if we consider 100 samples, most probably 0.27 of these may fall outside the $\mu \pm 3\sigma$ limits. So, if an observation falls outside the 3σ limits in 100 observations, it is logical to suspect that something might have gone wrong. Therefore, the control limits ona control chart are set up by using 3 limits. The **UCL and LCL of a control chart are called 3σ limits** of the chart. The question is: How do we calculate 3σ limits?

Suppose M is a sample statistic (e.g., mean, range, proportion of defectives, etc.) that measures some quality characteristic of interest. Further suppose that μ_M and σ_M are the mean and standard error (standard deviation) of the sample statistic M, respectively. Then the centre line and control limits for controllingthe quality characteristic are given by:

$$\text{Centre line (CL)} = \mu_M \quad \ldots (1)$$

$$\text{Upper control limit (UCL)} = \mu_M + 3\sigma_M \quad \ldots (2)$$

$$\text{Lower control limit (LCL)} = \mu_M - 3\sigma_M \quad \ldots (3)$$

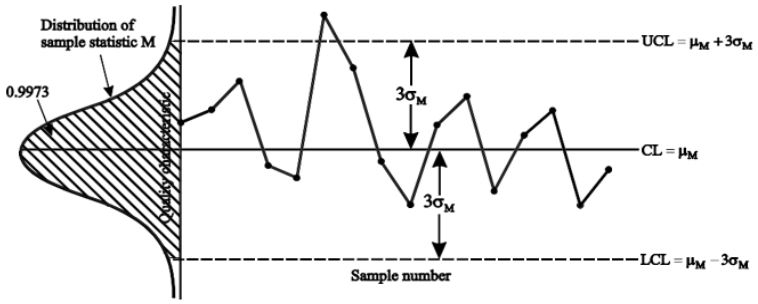

Fig. 1.2: 3σ limits.

The chart in Fig. 1.2 shows the centre line (CL), lower control limit (LCL) andupper control limit (UCL). The UCL and LCL are set at the distance ± $3\sigma_M$ from the centre line (μ_M). Note from Fig. 1.2 that the area covered between the$UCL(= \mu_M + 3\sigma_M)$ and $LCL(= \mu_M + 3\sigma_M)$ is 0.9973 (99.73%). So the probability that an observation falls outside these limits is 0.0027.

If the sample points fall between the control lines, the process is said to be under statistical control. But, if one or more points lie outside the control limits, control chart alarms (indicates) that the process is not under statistical control.

Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them and then continue the production process.

You may now like to check your understanding of 3σ limits by answering the following exercise.

E5) If μ and σ represent the mean and standard deviation of the process, the lower and upper three sigma control limits for a control chart are given by:

a) $\mu - 3\sigma^2$ *and* $\mu + 3\sigma^2$

b) $\mu - 3\sigma$ and $\mu + 3\sigma$

c) $\mu^2 - 3\sigma^2$ *and* $\mu^2 + 3\sigma^2$

d) $\mu^2 - 3\sigma$ and $\mu^2 + 3\sigma$

You learned in Section 1.6 that the points on the control chart reflect a pattern. If one or more sample points are beyond the control limits, the process is said to be out-of-control; if all sample points are inside or near the control limits, the process is said to be statistically controlled. However, in many circumstances, the sample points may be inside the control boundaries while exhibiting an unusual or particular pattern. This also indicates the presence of assignable causes. To determine if the process is statistically controlled or not, the pattern of the sample points must be examined. This is covered in Section 1.7.

1.7 Control Chart Patterns

The patterns of the control chart are broadly classified into two categories:

1. Natural patterns of variation, and

2. Unnatural patterns of variation.

We first discuss the natural patterns of variation.

1.7.1 Natural Patterns of Variations

In Unit 1 of MST-002 entitled Analysis of Quantitative Data, you have studied that the central tendency is the characteristic of the distribution. Most observations tend to concentrate near the centre (mean) of the distribution and very few points lie near the tails. Since normal distribution is symmetrical about its mean (μ), i.e., the centre line is at μ (CL = μ), we expect that half the

points will lie above the centre line and half below it. We also know that for the normal distribution

$$\Pi[\mu - 3\sigma \leq \Xi \leq \mu + 3\sigma] = 0.9973$$

This means that 99.73% observations lie between the 3σ limits. So, of a total of 100 observations, 99.73 observations will lie inside the 3σ limits and only

0.27 observations may lie outside the 3σ limits.

We may conclude that a control chart having a natural pattern of variation has the following three characteristics:

i) Most points lie near the centre line of the chart.

ii) Very few points lie near the control limits.

iii) None of the points fall outside the control limits.

A typical control chart with 3σ limits is shown in Fig.1.3

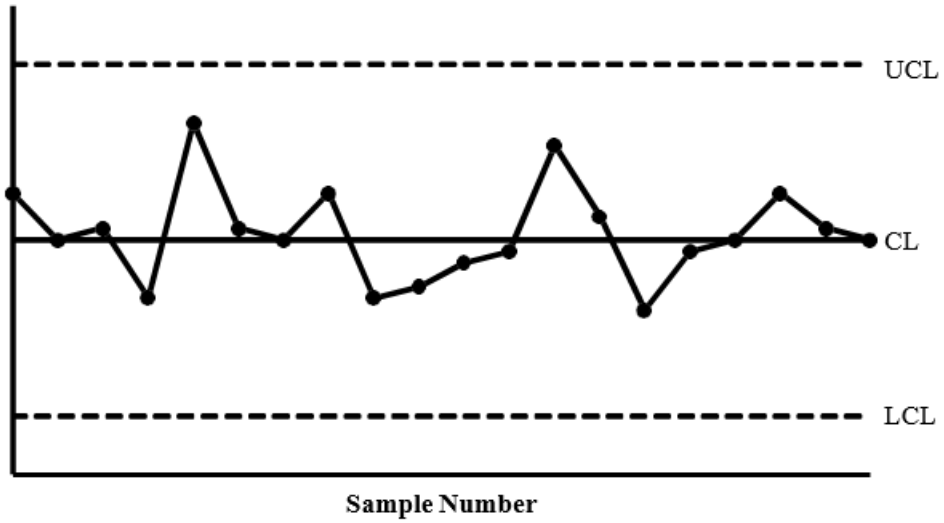

Fig. 1.3: Typical control chart.

1.7.2 Unnatural patterns of Variations

Engineers of Western Electric Company discovered 15 abnormal common patterns in control charts. In this part, we will look at the most common sorts of strange patterns:

i. **Extreme Variations:** If one or more sample points deviate considerably from the others and go outside the control bounds of the control chart, as seen in Fig.1.4, we state that the chart has severe variation. The process has certain assignable causes, and corrective action is required to bring the process under control.

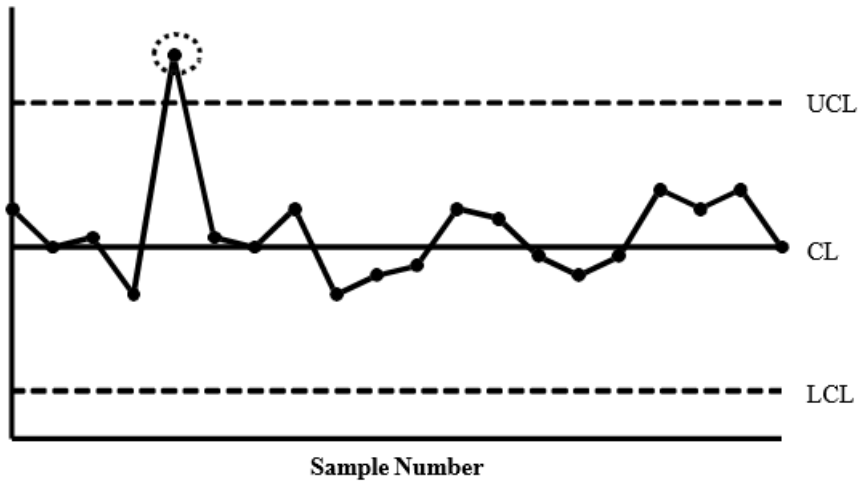

Fig. 1.4: Extreme variation.

Causes of extreme variation are:

i) Error in measurement, recording and calculations,
ii) Wrong setting and defective machine tools or erroneous use,
iii) Power failures for short time,

iv) Use of a new tool, failure of the component at the time of test, etc.

A sample point falling outside the control limits is a clear indication of the presence of assignable causes. There are other situations where in the pattern of sample points on the chart indicates the presence of assignable causes, although all points may lie within the control limits. Such situations are discussed below.

ii. **Trend:** If successive points on the control chart tend to go higher or lower, as illustrated in Fig. 1.5, the process reveals a trend. The process may spiral out of control if sufficient care or remedial action is not taken.

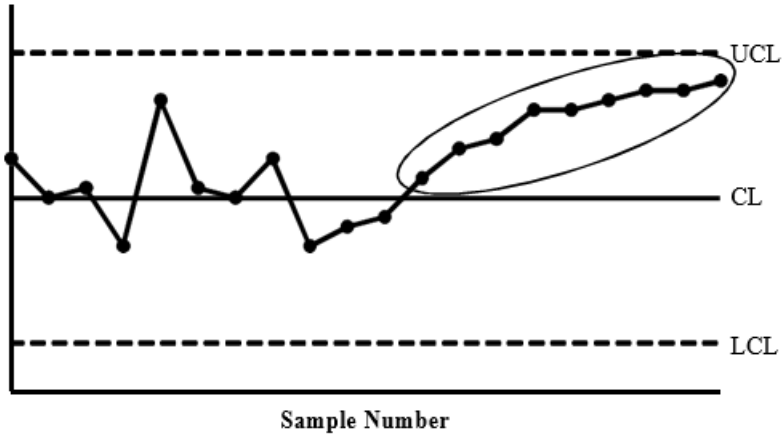

Fig. 1.5: Trend pattern.

Causes of trend pattern are:

i) Tool or die wear,
ii) Gradual change in temperature or humidity,
iii) Gradual wearing of operating machine parts,
iv) Gradual deterioration of equipment, etc.

iii. **Cycles:** When consecutive points display a cyclic pattern (as illustrated in Fig. 1.6), it indicates that assignable causes are present in the process and impact it on a regular basis.

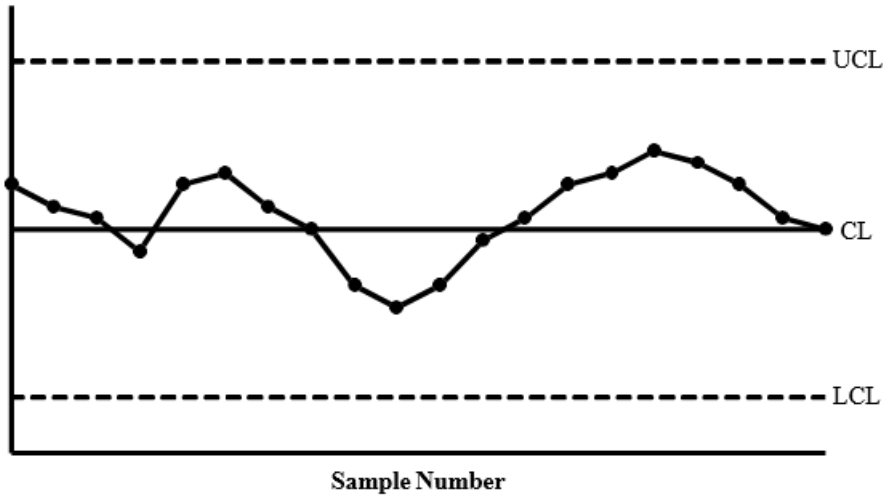

Fig. 1.6: Cyclic pattern.

Causes of cyclic pattern are:

i) Rotation of operators,
ii) Periodic changes in temperature and humidity,
iii) Periodicity in the mechanical or chemical properties of the material,
iv) Seasonal variation of incoming component, etc

iv. **Shifts:** When a string of successive points falls above or below the chart's centre line, it is thought that a change in the process has occurred. This suggests the presence of assignable causes. Figure 1.7 depicts such a design.

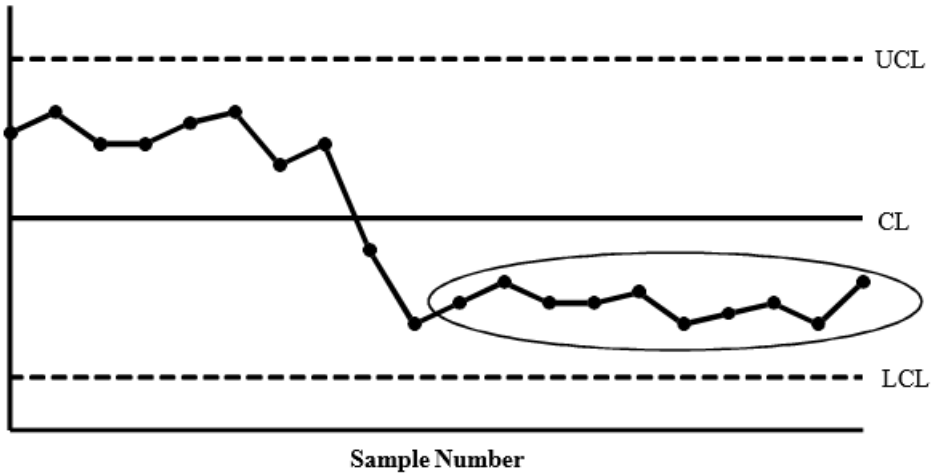

Fig. 1.7: Shift pattern.

Generally, it is assumed that when 7 consecutive points lie above or below the centre line, a shift has occurred.

Causes of shift pattern are:

i) Change in material,
ii) Change in machine setting,
iii) Change in operator, inspector, inspection equipment, etc.

v. Erratic Function: When the control chart sample points tend to fall close or somewhat beyond the control boundaries, with relatively few points near the centre line, as illustrated in Fig. 1.8, unpredictable fluctuation has occurred. This suggests the presence of assignable causes. The causes of unpredictable variations are quite difficult to pinpoint. These might be the result of different factors operating at different points in the process.

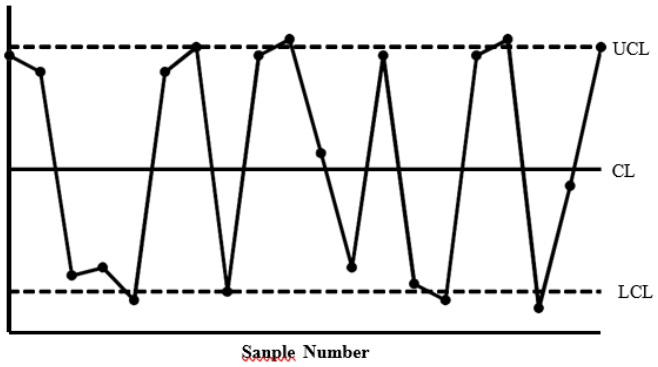

Fig. 1.8: Erratic fluctuations.

Causes of erratic fluctuations are:

i) Different types of materials being processed,
ii) Change in operator, machine, inspection, equipment,
iii) Frequent adjustment of machine, etc.

You may now like to review what you have studied about control charts patterns.

E6) Choose the correct option from the following:

i) If the points on the chart have continuous movements upward/downward, the pattern is called

a) freak pattern

b) shift pattern

c) trend pattern

d) cyclic pattern

ii) If all points on the control chart are within the control limits and the pattern of the points show trend, the process is said to be

a) under control

b) out-of-control

c) both (a) and (b)

iii) To check whether the process is under control or out-of-control, we see

a) the pattern of the sample points on the chart

b) the position of the sample points on the control chart

c) both (a) and (b).

1.8 Advantages and Limitation of SQC

When a large number of items/units are produced, the producer has two options for ensuring the quality of the lot: first, he or she can inspect each and every item and decide on the quality of the product, i.e., 100% inspection. Second, he or she may employ statistical quality control procedures, which entail inspecting a limited number of objects and deciding on the quality of the entire lot of manufactured goods. SQC provides a number of advantages to 100% inspections, which are detailed below.

i. **Ease of Application:** The ease of use of statistical quality control is an outstanding feature. Skilled and competent individuals are essential when creating statistical approaches for quality control. However, even individuals without substantial specialised expertise may easily employ statistical approaches.
ii. **Reduction in Cost:** The cost of inspection is reduced. In SQC as only a part or fraction of a lot is taken and inspected.
iii. **Greater Efficiency:** The inspection of each item is bound to diminish the effectiveness of quality control inspectors due to boredom. Inspectors are more vigilant while using. SQC since just a portion of the product is examined.
iv. **Early Detection of Faulty Units:** SQC is the process of continuously verifying the product's quality. When a sample point slips outside the control bounds, it indicates that the process is not statistically controlled. If there are any assignable reasons in the process, corrective action can be done. As a result, SQC assures early discovery of flaws and reduces item waste.
v. **Helpful in Specification:** Using SQC, we may determine whether or not the manufactured item is under control, that is, whether or not the item fulfils the specifications within the tolerance limits. If the variance exceeds the tolerance limits, SQC sends a warning signal, allowing remedial action to be done. So, as long as statistical control is maintained, future specifications may be reliably anticipated, which cannot be guaranteed by 100% inspection.
vi. **Ensures Overall Coordination's:** SQC techniques ensure collaboration among managers responsible for specifications, manufacturing, and inspection. It serves as a foundation for resolving conflicts among an organization's varied interests.
vii. **Determination of the effect of change in the process:** With the help of control charts, we can easily detect whether or not a change in the production process results in a significant change in the quality.
viii. **Equilibrium in Consumer's and Producer's Risk:** Methods such as quality control and acceptance sampling help in maintaining equilibrium between the consumer's risk and producer's risk.
ix. **Wider Application:** It is important not just for examining goods made in tiny quantities, but also for large manufacturing.
x. **Unique Method:** Statistical quality control is useful for things that are destroyed when inspected for a certain quality feature, such as the intensity of match sticks, the average life of a compact fluorescent lamp (CFL), the strength of glass, and so on. In such circumstances, 100% examination will ruin the entire lot and result in a significant loss.

However, SQC has some limitations, which are described below:

1. When a sample of the items drawn from the lot is not a true representative of the entire lot, does not have the same characteristics as the lot from which it is drawn. Then a good lot may be rejected and a bad one may be accepted. This is the main limitation of SQC.

2. SQC cannot be used mechanically for any production process without studying the process and without adequate knowledge about it.

3. SQC applied on a production process provides only the information that the process is under control or out-of-control. It cannot take any action for improvement.

1.9 Solutions / Answers

E1) i) Option (d) is the correct option because we know that quality has to incorporate the following:

- conforming to specifications,
- fitness for use,
- customer satisfaction,
- delighting the customer, and
- enchanting the customers.

ii) Option (c) is the correct option because we know that the performance is the primary operating characteristics of a product. Features are the additional characteristics available in product along with the primary operating characteristics. Reliability refers to the probability of a product's failure within a specified time period. Aesthetics means how a product looks, sounds, feels, etc.

iii) Option (a) is the correct option because we know that durability means a measure of product life. Conformance means meeting the specifications or standard. Reputation is related to the past performance of the company. Reliability refers to the probability of a product's failure within a specified time period.

E2) i) Option (c) is the correct option because we know that statistical quality control is a technique of both process control and product control.

ii) Option (c) is the correct option because we know that statistical quality control is a technique of both process control and product control. The control charts are used for process control and acceptance sampling plans are used for product control.

iii) Option (a) is the correct option because we know that process control is achieved through control charts, whereas product control is achieved through acceptance sampling plans.

iv) Option (b) is the correct option because we know that product control is achieved through acceptance sampling plans, whereasprocess control is achieved through control charts.

E3) i) Option (c) is the correct option because we know that the variation in quality characteristic may be due to assignable causes and chance causes.

ii) Option (b) is the correct option because we know that the variation due to chance causes is not controllable whereas the variation due to assignable causes is controllable.

iii) Option (a) is the correct option because we know that the variation due to assignable causes is controllable whereas the variation due to chance causes is not controllable.

iv) Option (d) is the correct option because we know that chance causes affect the process output in minor ways.

v) Option (a) is the correct option because we know that the variation due to assignable causes can be removed always.

E4) i) Option (b) is the correct option because we know that the control chart is a two-dimensional graphical display of a quality characteristic.

ii) Option (c) is the correct option because we know that the control chart consists of the centre line, upper control line, and lower control line.

iii) Option (c) is the correct option because we know that if one or more points lie outside the control limits, the process is not under statistical control. Some assignable causes are present in the process.

iv) Option (c) is the correct option because we know that control charts in statistical quality control are used for describing the pattern of variation and checking whether the variability in the product is within the tolerance limits or not.

E5) Option (b) is the correct option because we know that the lower and upper three sigma control limits for a control chart are $\mu \pm 3\sigma$ where μ and σ are the mean and the standard deviation of a random variable.

E6) i) Option (c) is the correct option because we know that if some consecutive points on the control chart tend to move upward or downward, it is called a trend pattern. If one sample point is significantly different from the other points and lies outside the control limits of the control chart, we say that there is an extreme variation (freak) in the chart. If a series of consecutive points falls above or below the centre line of the chart, it can be assumed that a shift in the process has taken place. If points on the chart have peaks and troughs which repeat themselves, we say that there is a cyclic pattern. These patterns are also an indication of the assignable causes.

ii) Option (b) is the correct option because we know that if the sample points show trend, it is also an indication of assignable causes and the process is out-of-control.

iii) Option (c) is the correct option because for checking whether the process is under control or out-of-control, we see the pattern of the sample points on the chart as well as the position of the sample points on the control chart.